

BLIND SIGNAL SEPARATION BY COMBINING TWO ICA ALGORITHMS: HOS-BASED EFICA AND TIME STRUCTURE-BASED WASOBI

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ABSTRACT

The aim of this paper is to combine the strengths of two recently proposed Blind Source Separation (BSS) algorithms. The first algorithm, abbreviated as EFICA, is a sophisticated variant of the well-known Independent Component Analysis (ICA) algorithm FastICA. EFICA is based on minimizing the statistical dependencies between the instantaneous (marginal) distributions of the estimated source signals and therefore disregards any possible time structure of the sources. The second algorithm, WASOBI, is a weight-adjusted variant of SOBI, a popular BSS algorithm that uses only the time structure of the source signals to achieve the separation. The separation accuracy of EFICA and WASOBI can be assessed using the estimated source signals alone, therefore allowing us to choose the most appropriate of the two in every scenario. Here, two different EFICA-WASOBI combination approaches are proposed and their performance assessed using images and simulated signals.

1. INTRODUCTION

Blind Source Separation (BSS) is a problem which has been widely studied in the last two decades. It consists in recovering a set of unknown source signals from their observed mixtures making as few assumptions about the mixing process as possible. The linear and instantaneous BSS model can be formulated as follows

$$\mathbf{x} = \mathbf{A}\mathbf{s} \quad (1)$$

where \mathbf{s} represents a $d \times N$ data matrix having as rows the unobserved source signals \mathbf{s}_k , $k = 1, \dots, d$. In this paper we will consider only the square, invertible case, where the number of observed mixtures is equal to the number of sources. The goal of BSS techniques is to estimate a separating matrix $\widehat{\mathbf{W}}$ such that $\widehat{\mathbf{s}} = \widehat{\mathbf{W}}\mathbf{x} \approx \mathbf{P}\mathbf{D}\mathbf{s}$ where \mathbf{D} is a diagonal matrix and \mathbf{P} is a permutation matrix. If the source signals are assumed to be mutually independent, the BSS problem can be solved by recovering independence in the estimated source signals, i.e. by using Independent Component Analysis [1].

Without any loss of generality we can assume that the source signals are centered (have zero mean and unit

variance). As we have mentioned above neither the order nor the scale of the source signals can be recovered. However, in order to simplify the derivations in this paper we will assume that the scale of the estimated sources is the correct one and we use the re-ordering proposed in [2] to guarantee that if the sources are perfectly recovered then $\widehat{\mathbf{s}} = \mathbf{s}$ and $\mathbf{G} = \widehat{\mathbf{W}}\mathbf{A} = \mathbf{I}$, where \mathbf{I} denotes the $d \times d$ identity matrix.

For an estimate of the de-mixing matrix $\widehat{\mathbf{W}}$ the Interference-to-Signal Ratio (ISR) matrix is defined by

$$\mathbf{ISR}_{k\ell} = \frac{\mathbf{G}_{k\ell}^2}{\mathbf{G}_{kk}^2}, \quad k, \ell = 1, 2, \dots, d. \quad (2)$$

The ISR of the k -th estimated signal is the k -th element of a d -dimensional vector \mathbf{isr} , where

$$\mathbf{isr}_k = \frac{\sum_{\ell=1, \ell \neq k}^d \mathbf{G}_{k\ell}^2}{\mathbf{G}_{kk}^2} \quad k = 1, 2, \dots, d. \quad (3)$$

At least three major types of separation criteria have been proposed in the literature based on modeling the source signals either as 1) non-Gaussian independent and identically distributed (i.i.d.) processes [3–6], 2) weakly stationary (WS) random processes driven by white Gaussian noise [7, 8], or 3) sequences of independent Gaussian variables with time-varying variances [9]. However, real-life data is not expected to fit perfectly in any of the models above but rather a combination of the three of them. For instance, electroencephalographic (EEG) signals are known to have strong temporal and spatial structure which suggest that they could be better modeled by a suitable combination of models 1) and 2) above. Several attempts of combining those two models can be found in the literature (see e.g. [10, 11]). They are usually based on defining new separation criteria by adding together a time-structure-based criterion and a spatial-structure-based criterion. Theoretically, a more effective combination could be obtained by modeling the signals as WS processes driven by arbitrarily distributed white noise. However, derivation of an efficient algorithm for such model is a very difficult task. The approach that we take here is somewhat different and is based on choosing for each source signal the best fitting model among models 1) and 2).

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The actual BSS algorithms that we aim to combine are EFICA [6] and WASOBI [8, 12, 14], which are two recently proposed algorithms, shown to be asymptotically efficient within the scope of model 1) and model 2), respectively [13, 15].

2. EFICA

EFICA is an ICA algorithm designed to separate non-Gaussian i.i.d. signals. The underlying assumption is that each source signal \mathbf{s}_k , $k = 1, \dots, d$ consists of N independent realizations of a random variable ξ_k having a non-Gaussian distribution function $F_k(x) = P(\xi_k \leq x)$ ¹.

The algorithm EFICA is a version of the FastICA [4] algorithm that features adaptive choice of the FastICA non-linearity. Let $g_k(\cdot)$ be the nonlinear function chosen for k -th signal, $k = 1, \dots, d$ and let $g'_k(\cdot)$ be its derivative. Finally, let “E” stand for the expectation operator, which can be realized by the sample mean. Then, the elements of the ISR matrix are asymptotically equal to [6]

$$\mathbf{ISR}_{k\ell}^{EF} = \frac{1}{N} \frac{\gamma_k(\gamma_\ell + \tau_\ell^2)}{\tau_\ell^2 \gamma_k + \tau_k^2(\gamma_\ell + \tau_\ell^2)} \quad (4)$$

where

$$\begin{aligned} \gamma_k &= \beta_k - \mu_k^2 & \mu_k &= \mathbb{E}[\widehat{\mathbf{s}}_k g_k(\widehat{\mathbf{s}}_k)] \\ \tau_k &= |\mu_k - \rho_k| & \rho_k &= \mathbb{E}[g'_k(\widehat{\mathbf{s}}_k)] \\ & & \beta_k &= \mathbb{E}[g_k^2(\widehat{\mathbf{s}}_k)] \end{aligned}$$

In the best possible case, i.e., when g_k equals the score function ψ_k of the corresponding distribution F_k (if it exists) for all $k = 1, \dots, d$, then equation (4) is equal to the corresponding Cramer-Rao Lower Bound (CRLB) [15], which is

$$\mathbf{CRLB}_{k\ell} = \frac{1}{N} \frac{\kappa_\ell}{\kappa_k \kappa_\ell - 1}, \quad (5)$$

where $\kappa_k = \mathbb{E}[\psi_k^2(\mathbf{s}_k)]$.

The theoretical ISR was shown to approximate the empirical ISR very well provided that the independent components are i.i.d., that means that they have no time structure. If the components are strongly auto-correlated, the theoretical ISR appears to be biased, in particular, overly optimistic.

3. WASOBI

WASOBI is a second-order BSS algorithm that exploits the time structure of the sources. The time-lagged sample correlation matrices of the observed mixtures

$$\mathbf{R}_x[\tau] = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n + \tau] \quad \tau = 0, \dots, M - 1 \quad (6)$$

(where $\mathbf{x}[n]$ denotes the n -th column of the $d \times N$ matrix \mathbf{x}) are related to the time-shifted sample correlation matrices of the sources by

$$\mathbf{R}_x[\tau] = \mathbf{A} \mathbf{R}_s[\tau] \mathbf{A}^T \quad \tau = 0, \dots, M - 1 \quad (7)$$

¹To be exact, at most one of the independent components is allowed to have Gaussian distribution

where due to the spatial decorrelation of the sources, their correlation matrices $\mathbf{R}_s[\tau] = \text{diag}[R_1[\tau], R_2[\tau], \dots, R_d[\tau]]^T$ are diagonal and $R_k[\tau]$ is the auto-correlation of $s_k[n]$ at lag τ . The original SOBI algorithm estimates matrix \mathbf{A} by enforcing perfect diagonalization of $\mathbf{R}_x[0]$ and approximate unitary joint diagonalization of $\mathbf{R}_x[\ell]$ for $\ell = 1, \dots, M - 1$. This operation implies sub-optimal weighting of the errors in the correlation estimates [8].

In WASOBI, the problem of diagonalizing $\mathbf{R}_x[\ell]$ for $\ell = 0, \dots, M - 1$ is reformulated as a non-linear weighted least squares (WLS) problem. An asymptotically optimal weight matrix can be obtained for the case of Auto-Regressive (AR) Gaussian sources. If all source signals are Gaussian AR processes of order $M - 1$ the asymptotic ISR matrix of the WASOBI estimates is equal to the corresponding CRLB [12]:

$$\mathbf{ISR}_{k\ell}^{WA} = \mathbf{CRLB}_{k\ell} = \frac{1}{N} \frac{\phi_{k\ell}}{1 - \phi_{k\ell} \phi_{\ell k}} \frac{\sigma_k^2 R_\ell[0]}{\sigma_\ell^2 R_k[0]} \quad (8)$$

where σ_k^2 is the variance of the innovation sequence of the k -th source,

$$\phi_{k\ell} \triangleq \frac{1}{\sigma_k^2} \sum_{i,j=0}^{M-1} a_{i\ell} a_{j\ell} R_k[i - j]$$

and $\{a_{i\ell}\}_{i=0}^{M-1}$ are AR coefficients of the ℓ -th source with $a_{0\ell} = 1$ for $k, \ell = 1, \dots, d$.

4. PROPOSED METHODS

The major reason for using WASOBI and EFICA to combine temporal and spatial information in the separation process is that their theoretical performance can be estimated via (4) and (8) using consistent estimates of the statistical quantities involved, i.e., sample means and Yule-Walker or other AR coefficients estimates, respectively. This suggests a computationally appealing means for evaluating (at least asymptotically) the accuracy of the estimated components. Indeed the expressions in (4) and (8) are theoretically valid only for their corresponding assumed signal model. Nevertheless, simulation experiments show that they are robust enough to be considered valid even when their assumed model is only approximately obeyed. An alternative method for assessing the separation accuracy of BSS algorithms is to use bootstrap methods [16]. However, this latter approach has the major drawback of being computationally very demanding. Furthermore, the definition of bootstrap surrogates of data with time structure is not trivial. Another reason for choosing EFICA and WASOBI is that they are asymptotically efficient for their respective signal models which envisages good performance of a combination of the two algorithms.

A straightforward approach to combine EFICA and WASOBI would be to apply a simple per-signal decision-based method, namely to decide between signals estimated via EFICA and WASOBI by comparing (4) and (8) (for each k). However, the main drawback of such an approach is that does not eliminate the effect of the “inappropriate” sources for each algorithm. In other words,

in an ideal situation it would be desired to first separate the mixtures into two isolated groups: one consisting only of mixtures of the i.i.d. non-Gaussian sources (optimally separable by EFICA), and the other consisting of the remaining sources (optimally separable by WASOBI). Then each group would be separated using its respective algorithm, thus eliminating upfront the effect of “nuisance sources” for each algorithm. However, since this initial isolation stage is not feasible (and, moreover, not each source can be labeled as belonging exclusively to one of the two groups), we propose two alternative approaches, which work iteratively to approach the desired isolation, combining the strengths of both EFICA and WASOBI:

4.1 Algorithm EFWS

The first method, called EFWS (from EFica-WaSobi), proceeds in two main steps:

1. (a) Using EFICA, estimate the sources \mathbf{s}^{EF} from the mixtures \mathbf{x} .
- (b) Estimate the ISR matrix achieved by EFICA, \mathbf{ISR}^{EF} via (4) and the corresponding vector \mathbf{isr}^{EF} .
- (c) Incorporating \mathbf{s}^{EF} into formula (8), compute the asymptotical ISR matrix achieved by WASOBI \mathbf{ISR}^{WA} and the corresponding vector \mathbf{isr}^{WA} .
2. For each $k = 1, \dots, d$ accept the estimated signal \mathbf{s}_k^{EF} iff $\mathbf{isr}_k^{EF} > \mathbf{isr}_k^{WA}$. Let the accepted and the rejected signals be denoted by \mathbf{u} and \mathbf{v} , respectively. Then, apply algorithm WASOBI to the rejected signals \mathbf{v} .

4.2 Algorithm COMBI

The second method, abbreviated as COMBI, is more sophisticated method than EFWS at the expense of higher computational requirements. It proceeds in the following steps:

1. Let $\mathbf{z} = \mathbf{x}$
2. Apply both algorithms EFICA and WASOBI to \mathbf{z} ; let the estimated source signals be \mathbf{s}^{EF} and \mathbf{s}^{WA} , respectively. Similarly, the estimated ISR matrix are \mathbf{ISR}^{EF} and \mathbf{ISR}^{WA} , and the corresponding vectors \mathbf{isr}^{EF} and \mathbf{isr}^{WA} .
3. Let $E = \min_k \mathbf{isr}_k^{EF}$ and $W = \min_k \mathbf{isr}_k^{WA}$
4. If $E < W$,
 - (a) accept those signals \mathbf{s}^{EF} for which $\mathbf{isr}_k^{EF} < W$ and redefine \mathbf{z} as the rejected signals of \mathbf{s}^{EF} .
 - else,
 - (b) accept those signals \mathbf{s}^{WA} for which $\mathbf{isr}_k^{WA} < E$ and redefine \mathbf{z} as the rejected signals of \mathbf{s}^{WA}
5. If there are more than one rejected signals, go to (2). Otherwise, if any, accept the rejected signal.

5. SIMULATIONS

An illustrative comparison with well-known ICA algorithms [3,4,8,10,11] has been conducted to demonstrate the advantages of the proposed methods. Four signals of length $N = 1000$ samples were mixed using a random matrix. The first two source signals (denoted by AR1

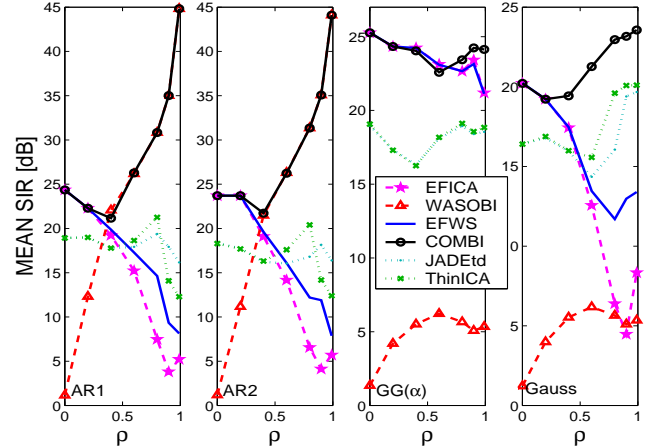


Figure 1: SIR for sources AR1, AR2, $GG(\alpha)$ and Gauss when $\alpha = 1$ and $\rho \in [0, 1)$. The shown SIR values are the average from 100 random realizations of the mixing matrix and the sources.

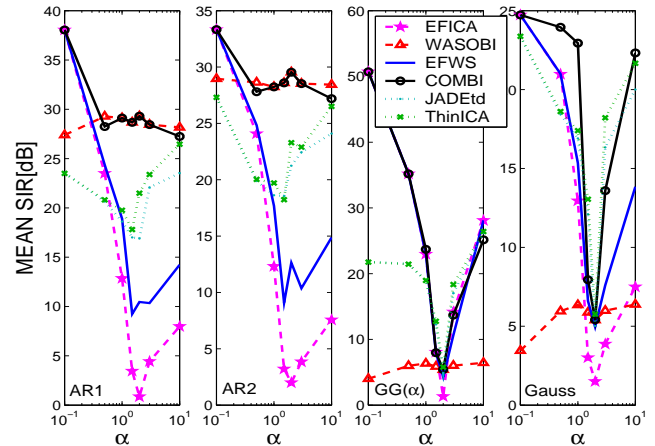


Figure 2: SIR for sources AR1, AR2, $GG(\alpha)$ and Gauss when $\rho = 0.7$ and $\alpha \in [0.1, 10]$. The shown SIR values are the average from 100 random realizations of the mixing matrix and the sources.

and AR2 in figures 1 and 2) were AR processes generated from white noise having a generalized Gaussian distribution $GG(\alpha)$ - for definition see, e.g., Appendix B in [6]. The AR coefficients of sources AR1 and AR2 were $(1, \rho)$ and $(1, -\rho)$, respectively. The third source was an i.i.d. process having $GG(\alpha)$ distribution (this source is denoted by $GG(\alpha)$ in the figures). The last source was a Gaussian white noise signal (denoted by Gauss). The separation performance was measured in terms of Signal-To-Interference ratio (SIR) which is just the inverse of the ISR defined in equation (3).

Note that for “small” values of ρ the signals have no time structure (i.e., they are i.i.d. in time) while the divergence from Gaussianity grows with α being far from 2, which is a scenario much more suitable for EFICA than for WASOBI. By contrast, for $\rho \rightarrow 1$ the time structure of the two AR processes becomes predominant over their non-Gaussianity. In this latter

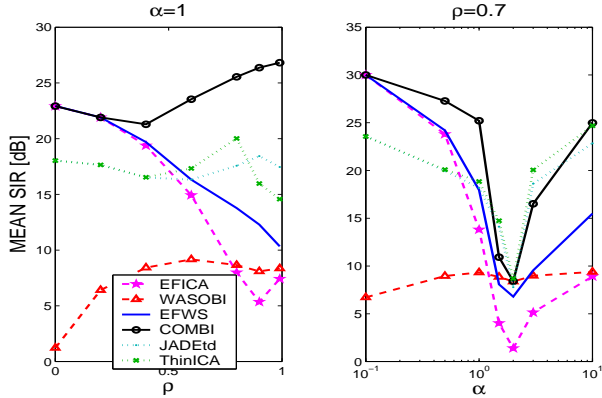


Figure 3: Mean SIR (average of individual signals SIRs) for a fixed $\alpha = 1$ and a fixed $\rho = 0.7$, respectively. The shown values are the average results for 100 random realizations of the mixing matrix and the sources.

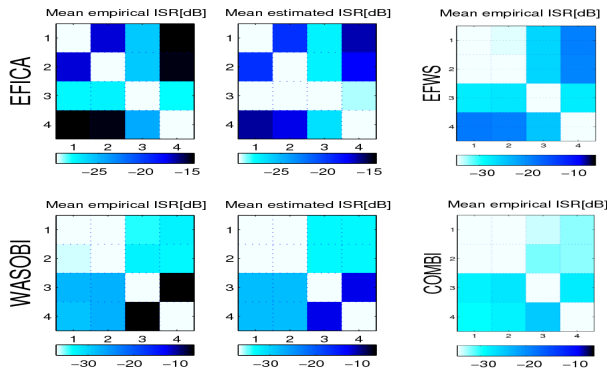


Figure 4: Average empirical ISR matrices of EFICA, WASOBI, EFWS and COMBI and average theoretical ISR matrices of EFICA, WASOBI achieved in separation of the source signals AR1, AR2, GG(α), and Gauss, with $\alpha = 1$ and $\rho = 0.7$.

case, WASOBI outperforms EFICA as is shown in figure 3. Both proposed methods COMBI and EFWS benefit from the complementary advantages of algorithms EFICA and WASOBI. Moreover, they are able to estimate the Gaussian i.i.d. signal, which cannot be separated unless all the other signals are correctly estimated.

Using 100 random repetitions of the sources (for $\alpha = 1$ and $\rho = 0.7$) and the mixing matrix we assessed the performance of the ISR estimators (4) and (8) by computing their mean values. The results are shown in figure 4, together with the mean of the true (empirical) ISR matrix. The results suggest that the proposed estimators are very accurate. The figure also includes the empirical ISR matrices achieved by the novel methods EFWS and COMBI.

To demonstrate the performance of the algorithm on real data we used a dataset consisting of the grayscale images shown in figure 7. The first two images were typical textures while the last two images were synthetically generated noise. These images were obtained from [17]. The size of the test images were 128×128 pixels and

were scanned column by column to form one dimensional source signals. After centering and normalization, the source images were mixed with a random mixing matrix, and subsequently separated using ICA. In order to study the effect of noise on the performance of the proposed algorithm we incorporated different levels of additive gaussian noise to the observed mixtures. The accuracy of the separation for the k^{th} source was assessed in terms of Signal-To-Interference-Plus-Noise Ratio (SINR) [18]:

$$\text{SINR}_k = \frac{\mathbf{G}_{kk}^2}{\sum_{i \neq k}^d \mathbf{G}_{ki}^2 + \sigma^2 \sum_{i=1}^d \mathbf{W}_{ki}^2} \quad (9)$$

where σ is the standard deviation of the noise. Note that the highest SINR is obtained when the mean squared difference between the true and estimated sources is minimum, i.e. when $\hat{\mathbf{W}} = \mathbf{A}^T (\mathbf{A}\mathbf{A}^T + \sigma^2 \mathbf{I})^{-1}$.

The average SINR obtained for test images 1 and 2, i.e. for the images with strong spatial autocorrelations, is shown in figure 5. The average SINR for the noise images (images 3 and 4) is shown in figure 6. From the results we can conclude that for low noise levels the accuracy of EFWS and especially of COMBI was clearly superior to the other tested algorithms. When the noise power becomes considerable the differences in the performance of the different algorithms become smaller but still EFWS and COMBI are among the best performing ones. The texture images have very strong spatial autocorrelations which explains the excellent accuracy of WASOBI in separating them. However, the noise images lack any autocorrelations, which makes them impossible to extract for WASOBI. The two proposed algorithms EFWS and COMBI are able to effectively combine the excellent performance of WASOBI in separating the textures and the good performance of EFICA in separating the two noise images. The average CPU time of EFWS and COMBI was 0.83 and 1.47 seconds respectively.² The other algorithms, EFICA, WASOBI, SOBI, FastICA, JADE [3], JADE_{TD} [11], and ThinICA [10] had average computation times 0.63, 0.22, 0.14, 0.17, 0.06, 0.20 and 0.87 seconds, respectively. Overall, we can conclude that EFWS and COMBI are the algorithms that achieve the best trade-off between accuracy and computational cost for this dataset.

6. CONCLUSIONS

In this paper we have proposed novel ICA algorithms that effectively combine two powerful ICA methods EFICA and WASOBI. The combination allows source-selective separation of mixtures in which each source is either an i.i.d. non-Gaussian sequence or a stationary Gaussian process. Their wider applicability and superior accuracy were demonstrated using simulated and real data.

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²Computed in Matlab 7 on a computer equipped with two 3GHz Intel Xeon CPU's, 2.5 GB of memory, and 4GB of swap.

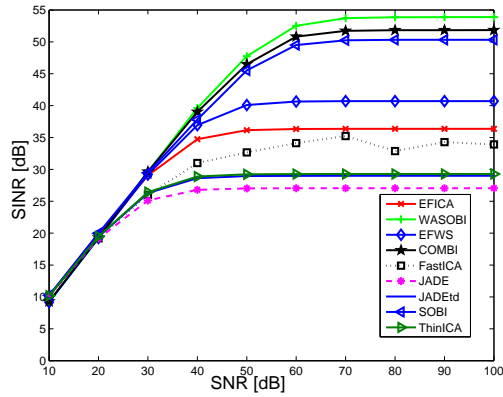


Figure 5: Average SINR of the estimates of test images 1 and 2 for different SNR levels of the observed mixtures.

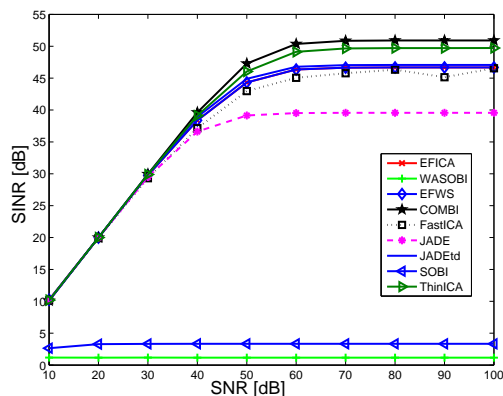


Figure 6: Average SINR of the estimates of test images 3 and 4 for different SNR levels of the observed mixtures.

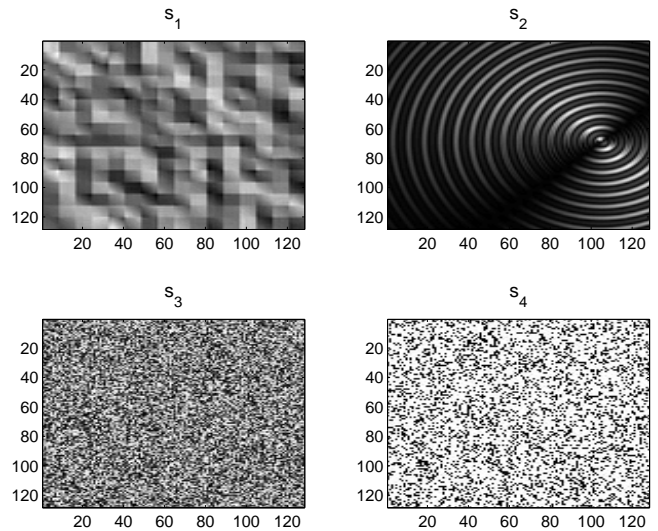


Figure 7: Test images

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